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Transient free convection flow past vertical cylinder with constant heat flux and mass transfer

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Abstract This paper describes a one dimensional unsteady natural convection flow past an infinite vertical cylinder with heat and mass transfer under the effect of constant heat flux at the surface of the cylinder. Closed form solutions of the dimensionless unsteady linear governing boundary layer equations are obtained in terms of Bessel functions and modified Bessel functions by Laplace transform method. The numerical values of velocity, temperature and concentration profiles are obtained for different values of the physical parameters namely, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and time and presented in graphs. Also, skin friction and Sherwood number are shown graphically and discussed. It is observed that the velocity and temperature increase unboundedly with time, while the concentration approaches steady state at larger times.

1. Introduction

Unsteady free convection flow of a viscous incompressible fluid over a heated vertical cylinder has attracted attention of many researchers because of their wide applications in the fields of engineering and geophysics such as nuclear reactor cooling system and underground energy system. In glass and polymer industries, hot filaments, which are considered as a vertical cylinder, are cooled as they pass through the surrounding environment. Goldstein and Briggs [1] analyzed the transient free convection about vertical plates and circular cylinders to a surrounding initially quiescent fluid. They presented analytical solutions for the infinite cylinders by Laplace transform method. Fujii and Uehara [2] compared laminar natural convection along the outer surface of a vertical cylinder with a vertical flat plate on heat transfer. Combined heat and mass transfer on natural convection along a vertical cylinder was studied by Chen and Yuh [3]. Their study covered a wide range of radii and Prandtl numbers. Velusamy and Garg [4] have given a numerical solution for the transient natural convection over a vertical cylinder of various thermal capacities and radii. A numerical solution for transient free convective flow over a vertical cylinder under the combined buoyancy effects of heat and mass transfer was investigated

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Nomenclature

C'	species concentration	R	dimensionless radial co-ordinate
C	dimensionless species concentration	Sc	Schmidt number
D	mass diffusion coefficient	Sh	Sherwood number
Gr	thermal Grashof number	t'	time
Gc	mass Grashof number	t	dimensionless time
g	acceleration due to gravity	T'	temperature
J_0	Bessel function of first kind and order zero	T	dimensionless temperature
J_1	Bessel function of first kind and order one	u	x-component of velocity
J_2	Bessel function of first kind and order two	U	dimensionless velocity along x-axis
k	the thermal conductivity	V	dummy real variable used in integral
K_0	modified Bessel function of second kind and order zero	Y_0	Bessel function of second kind and order zero
K_1	modified Bessel function of second kind and order one	Y_1	Bessel function of second kind and order one
Pr	Prandtl number	Y_2	Bessel function of second kind and order two
Q	the rate of heat supplied at the surface	α	thermal diffusivity of fluid
r	radial co-ordinate measured from the axis of the cylinder	ν	kinematic viscosity
		β	volumetric co-efficient of thermal expansion
		β^*	volumetric co-efficient of expansion with concentration

by Ganesan and Rani [5], while Ganesan and Rani [6] restudied for variable surface temperature. Yücel [7] studied numerically the natural convection over a vertical cylinder in a porous medium, while Hossain et al. [8] presented a numerical solution for flow past a vertical permeable cylinder in a non-Darcy porous medium. Also, Ganesan and Rani [9] studied magnetohydrodynamic effect on flow past a vertical cylinder with heat and mass transfer. Ganesan and Loganathan [10] analyzed heat and mass transfer on the unsteady natural convective flow past a moving vertical cylinder, numerically. They observed that there was a rise in the velocity due to the presence of mass diffusion. Thereafter, Ganesan and Loganathan [11] investigated the combined effects of radiation and chemical reaction by applying numerical method. Rani [12] studied the effects of variable surface temperature and concentration along a vertical cylinder. The boundary layer equations were solved by Crank–Nicolson type of implicit finite-difference method. Mohammed and Salman [13] studied combined convective flow past cylinder. Deka and Paul [14,15] have presented analytical solution of unsteady natural convective flow past an infinite vertical cylinder with constant surface temperature. They solved the governing boundary layer equations by Laplace transform method. Rashidi et al. [16] examined the free convective heat and mass transfer in a steady two-dimensional flow over a permeable vertical stretching sheet in the presence of radiation and buoyancy effects by homotopy analysis method. Machireddy [17] investigated the combined effects of chemical reaction and radiation on flow past cylinder, numerically. Makinde [18], Nandkeolyar et al. [19], Makinde and Tshehla [20] studied the unsteady hydromagnetic flow past vertical flat plate under different physical situations. Makinde [21] studied chemically reacting hydromagnetic unsteady flow of a radiating fluid past a vertical plate with constant heat flux. Again, Makinde [22] investigated the thermal boundary layer of nanofluids over an unsteady stretching sheet with a convective surface boundary condition using Runge–Kutta integration scheme with shooting technique, whereas Rashidi et al. [23] investigated the steady laminar incompressible free convective flow of a nanofluid past a chemically reacting fluid flowing upward facing horizontal plate in a porous medium with heat generation/absorption and the thermal slip boundary condition, wherein optimal homotopy analysis method (OHAM) was employed. Recently, Rashidi et al. [24] studied the buoyancy effect on MHD flow of a nanofluid over a stretching sheet in the presence of thermal radiation by employing shooting technique together with Runge–Kutta integration scheme.

The numerical solutions of convective flow past a vertical cylinder with constant heat and mass flux have been obtained by different authors, for example, Hess and Miller [25], Zariffeh and Daguenet [26], Hossain and Nakayama [27], Ganesan and Rani [28], Ganesan and Loganathan [29–31], Ishak [32] and Chinyoka and Makinde [33]. However, no exact solution on unsteady free convective flow past vertical cylinder with the combined effects of constant heat flux at the surface of the cylinder with mass transfer seems to have been reported and this motivates the present investigation. The unsteady non-dimensional linear governing equations are solved by the Laplace transform technique and employed complex inversion formula to find the inverses of the transformations. Also, attempt has been made to find solutions for larger times to observe for possible occurrence of steady state for different parameters, namely, thermal Grashof number, mass Grashof number and Schmidt number.

2. Mathematical formulation

We consider a unsteady, laminar and incompressible viscous flow past an infinite vertical cylinder of radius r_0. Here, the x-axis is taken along the axis of the cylinder in the vertically upward direction, and the radial co-ordinate r is taken normal to it. Initially, it is assumed that the cylinder and the fluid are at the same temperature T'_∞ and concentration near the surface is C'_∞. At time $t' > 0$, the concentration level near the cylinder is raised to C'_w and heat is supplied at a constant rate

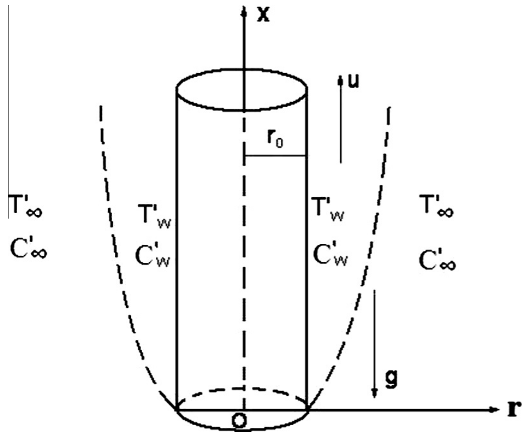

Figure 1 The physical model and co-ordinate system.

on the surface of the cylinder (see Fig. 1). With the Boussinesq's approximation, the governing boundary layer equations for vertical velocity (u), temperature (T') and concentration (C') for free convective flow (*cf.* Goldstein and Briggs [1], Chen and Yuh [3]) are as follows:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{g\beta}{v}(T'-T'_\infty)+\frac{g\beta^*}{v}(C'-C'_\infty)=\frac{1}{v}\frac{\partial u}{\partial t'} \quad (1)$$

$$\frac{\partial^2 T'}{\partial r^2}+\frac{1}{r}\frac{\partial T'}{\partial r}-\frac{1}{\alpha}\frac{\partial T'}{\partial t'}=0 \quad (2)$$

$$\frac{\partial^2 C'}{\partial r^2}+\frac{1}{r}\frac{\partial C'}{\partial r}-\frac{1}{D}\frac{\partial C'}{\partial t'}=0 \quad (3)$$

The initial and boundary conditions are,

$$\left.\begin{array}{l} u=0,\ \ T'=T'_\infty,\ \ C'=C'_\infty\ \ \forall r,\ \ t'\leqslant 0 \\ \left.\begin{array}{l} u=0,\ \ \frac{dT'}{dr}=-\frac{Q}{2\pi r_0 k},\ \ C'=C'_w \text{ at } r=r_0 \\ u\to 0,\ \ T'\to T'_\infty,\ \ C'\to C'_\infty \quad \text{as } r\to\infty \end{array}\right\} t'>0 \end{array}\right\} \quad (4)$$

We now introduce the following non-dimensional quantities:

$$\left.\begin{array}{l} R=\frac{r}{r_0},\ \ U=\frac{ur_0}{v},\ \ t=\frac{t'v}{r_0^2},\ \ T=\frac{2\pi k(T'-T'_\infty)}{Q},\ \ C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \\ Pr=\frac{v}{\alpha},\ \ Sc=\frac{v}{D},\ \ Gr=\frac{g\beta r_0^3 Q}{2\pi k v^2},\ \ Gc=\frac{g\beta^*(C'_w-C'_\infty)r_0^3}{v^2} \end{array}\right\} \quad (5)$$

Then the Eqs. (1)–(3) reduce to,

$$\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}+GrT+GcC=\frac{\partial U}{\partial t} \quad (6)$$

$$\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}-Pr\frac{\partial T}{\partial t}=0 \quad (7)$$

$$\frac{\partial^2 C}{\partial R^2}+\frac{1}{R}\frac{\partial C}{\partial R}-Sc\frac{\partial C}{\partial t}=0 \quad (8)$$

and the initial and boundary conditions (4) become,

$$\left.\begin{array}{l} U=0,\ \ T=0,\ \ C=0,\ \ \forall R,\ \ t\leqslant 0 \\ \left.\begin{array}{l} U=0,\ \ \frac{dT}{dR}=-1,\ \ C=1 \text{ at } R=1 \\ U\to 0,\ \ T\to 0,\ \ C\to 0 \quad \text{as } R\to\infty \end{array}\right\} t>0 \end{array}\right\} \quad (9)$$

3. Solution technique

Many different analytical methods have recently been introduced in the literature to solve non-linear ordinary and partial differential equations, namely; homotopy analysis method (HAM), variational iteration method (VIM), differential transform method (DTM) developed by Liao [34,35], He [36] and Zhou [37] respectively. In HAM series solutions are obtained and the convergence of the series is dependent on some base functions. On the other hand the series solutions obtained by DTM converge independently, if solution exists. He's VIM is based on the use of restricted variations, correction functional and Lagrange multiplier technique developed by Inokuti et al. [38]. This method does not require the presence of small parameters in the differential equation, and provides the solution as a sequence of iterates. Also, VIM does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives. This technique is, in fact, a modification of the general Lagrange multiplier method into an iteration method. In this technique, the equations are initially approximated with possible unknowns. A correction functional is established by the general Lagrange multiplier, which can be identified optimally via the variational theory. This method provides the convergent successive approximations of the exact solution, rapidly. Besides, the VIM is employed to solve other linear as well as non-linear problems in fractional calculus. In contrast, Laplace transform method is applied to solve only linear ordinary and partial differential equations yielding the solution expressions in closed form except some special cases. In particular, functions of periodic nature, of spike shapes, and whose Laplace transforms contain singularities, branch points are normally difficult to handle. In mathematical problems, namely in electromagnetics, flow past plates and cylinders, such types of functions exist. Then, one has to find inverse Laplace transforms of the types, namely; modified Bessel functions and exponential forms. In these cases, the inversions are obtained, in terms of series (*cf.* Ragab [39]), numerically (*cf.* Cheng and Sidauruk [40]) or in integral form (*cf.* Puri and Kythe [41]).

Here, we have applied Laplace transform technique to solve the non-dimensional governing Eqs. (6)–(8) subject to the initial and boundary conditions (9). The Laplace transforms of (6)–(8) subject to initial conditions in (9) give,

$$\frac{d^2\overline{U}}{dR^2}+\frac{1}{R}\frac{d\overline{U}}{dR}-s\overline{U}+Gr\overline{T}+Gc\overline{C}=0 \quad (10)$$

$$\frac{d^2\overline{T}}{dR^2}+\frac{1}{R}\frac{d\overline{T}}{dR}-sPr\overline{T}=0 \quad (11)$$

$$\frac{d^2\overline{C}}{dR^2}+\frac{1}{R}\frac{d\overline{C}}{dR}-sSc\overline{C}=0 \quad (12)$$

where s is the parameter of Laplace transform, defined by $L\{f(t)\}=F(s)$. $\overline{U}$, $\overline{T}$ and $\overline{C}$ are Laplace transforms of U, T and C respectively. The Eqs. (11) and (12) are solved for $\overline{T}$ and $\overline{C}$, taking the transformed boundary conditions into account, to get,

$$\overline{T}=\frac{K_0(\sqrt{sPr}R)}{s^{3/2}Pr^{1/2}K_1(\sqrt{sPr})} \quad (13)$$

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$$\overline{C} = \frac{K_0(\sqrt{sSc}R)}{sK_0(\sqrt{sSc})} \tag{14}$$

Now, using relations (13) and (14) in (10) and applying the transformed quantities of the boundary conditions on U, we obtain for $Pr \neq 1$, $Sc \neq 1$ and $Pr = Sc = 1$ respectively as,

$$\overline{U} = \frac{Gr}{s^{5/2}Pr^{1/2}(Pr-1)}\left[\frac{K_0(\sqrt{sPr})}{K_1(\sqrt{sPr})}\frac{K_0(\sqrt{s}R)}{K_0(\sqrt{s})} - \frac{K_0(\sqrt{sPr}R)}{K_0(\sqrt{sPr})}\right] + \frac{Gc}{s^2(Sc-1)}\left[\frac{K_0(\sqrt{s}R)}{K_0(\sqrt{s})} - \frac{K_0(\sqrt{sSc}R)}{K_0(\sqrt{sSc})}\right] \tag{15}$$

$$\overline{U} = \frac{Gr}{2s^2}\left[R\frac{K_1(\sqrt{s}R)}{K_1(\sqrt{s})} - \frac{K_0(\sqrt{s}R)}{K_0(\sqrt{s})}\right] + \frac{Gc}{2s^{3/2}}\left[R\frac{K_1(\sqrt{s}R)}{K_0(\sqrt{s})} - \frac{K_1(\sqrt{s})}{K_0(\sqrt{s})}\frac{K_0(\sqrt{s}R)}{K_0(\sqrt{s})}\right] \tag{16}$$

Now to determine the closed form of solution for T, C and U, we determine the inverses of (13)–(16) by applying complex inversion theorem. For definiteness, we present the procedure for determining C. We know by complex inversion theorem

$$C = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}\frac{K_0(R\sqrt{sSc})}{sK_0(\sqrt{sSc})}ds = \frac{1}{2\pi i}\oint_{C_1} e^{st}\frac{K_0(R\sqrt{sSc})}{sK_0(\sqrt{sSc})}ds \tag{17}$$

where C_1 is the Bromwich contour shown in Fig. 2.

The integrand of (17) has a branch point at $s = 0$ and a simple pole at $s = 0$, but $K_0(\sqrt{sSc})$ do not have zero at any point on the real and imaginary axis, if the branch cut is made along the negative real axis. The integral (17) may be replaced by the limit of the sum of the integrals over FE, ED, DC, CB and BA as $S_1 \to \infty$ and $S_0 \to 0$. The particular form of contour has been chosen because the values along the paths DC, BA and FE vanish as $S_1 \to \infty$ and $S_0 \to 0$.

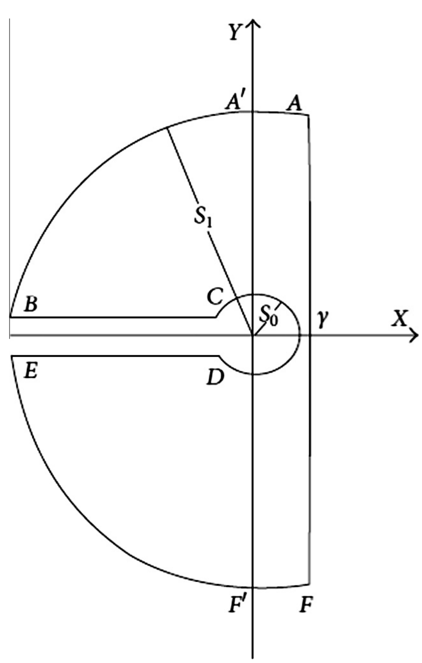

Figure 2 Bromwich contour.

Along the paths CB and ED, we chose $s = V^2e^{i\pi}/Sc$ and $s = V^2e^{-i\pi}/Sc$ respectively.

On the paths CB and ED, the integrals become,

$$I_{CB} = \frac{1}{\pi i}\int_0^\infty e^{-V^2t/Sc}\frac{J_0(RV) - iY_0(RV)}{J_0(V) - iY_0(V)}\frac{dV}{V} \tag{18}$$

and

$$I_{ED} = -\frac{1}{\pi i}\int_0^\infty e^{-V^2t/Sc}\frac{J_0(RV) + iY_0(RV)}{J_0(V) + iY_0(V)}\frac{dV}{V} \tag{19}$$

Also, the residue of the integrand of (17) at the simple pole at $s = 0$ is 1. Finally, we have

$$C = 1 + I_{CB} + I_{ED} = 1 + \frac{2}{\pi}\int_0^\infty e^{(-V^2t/Sc)}\frac{J_0(RV)Y_0(V) - Y_0(RV)J_0(V)}{J_0^2(V) + Y_0^2(V)}\frac{dV}{V} \tag{20}$$

which is rewritten as,

$$C = 1 + \frac{2}{\pi}\int_0^\infty e^{-V^2t/Sc}\Gamma_5(R,V)\frac{dV}{V} \tag{21}$$

Similarly, the inverse Laplace transforms of $\overline{T}$ and $\overline{U}$ can be obtained as,

$$T = -\frac{2}{\pi}\int_0^\infty (1 - e^{-V^2t/Pr})\Gamma_4(R,V)\frac{dV}{V^2} \tag{22}$$

$$U = \frac{2Gr}{(Pr-1)\pi}\int_0^\infty [V^2t + Pr(e^{-V^2t/Pr} - 1)]\xi(R,V,Pr)\frac{dV}{V^4} + \frac{2GcSc}{(Sc-1)\pi}\int_0^\infty (1 - e^{-V^2t/Sc})\Gamma_3(R,V,Sc)\frac{dV}{V^3}$$

for $Pr \neq 1$ and $Sc \neq 1$ (23)

$$U = \frac{Gr}{\pi}\int_0^\infty (1 - e^{-V^2t})\Gamma_1(R,V)\frac{dV}{V^3} + \frac{Gc}{\pi}\int_0^\infty (1 - e^{-V^2t})\Gamma_2(R,V)\frac{dV}{V^2}, \text{ for } Pr = Sc = 1 \tag{24}$$

Non-dimensional skin friction $\tau = -\frac{\partial U}{\partial R}\Big]_{R=1}$ can be obtained from the Eqs. (23) and (24) for $Pr \neq 1$, $Sc \neq 1$ and $Pr = Sc = 1$ respectively as,

$$\tau = \frac{2Gr}{(Pr-1)\pi\sqrt{Pr}}\int_0^\infty [V^2t + Pr(e^{-V^2t/Pr} - 1)]\Gamma_6(V,Pr)\frac{dV}{V^3} + \frac{2GcSc}{(Sc-1)\pi}\int_0^\infty (1 - e^{-V^2t/Sc})\Gamma_7(V,Sc)\frac{dV}{V^2} \tag{25}$$

$$\tau = \frac{Gr}{\pi}\int_0^\infty (1 - e^{-V^2t})\Gamma_9(V)\frac{dV}{V^2} + \frac{Gc}{\pi}\int_0^\infty (1 - e^{-V^2t})\Gamma_{10}(V)\frac{dV}{V^2} \tag{26}$$

The non-dimensional Sherwood number $Sh = -\frac{\partial C}{\partial R}\Big]_{R=1}$ can be obtained from the Eq. (21) as,

$$Sh = \frac{2}{\pi}\int_0^\infty e^{(-V^2t/Sc)}\Gamma_8(V)dV \tag{27}$$

where

$$\Gamma_1(R,V) = R\frac{J_1(RV)Y_1(V) - Y_1(RV)J_1(V)}{J_1^2(V) + Y_1^2(V)} - \Gamma_5(R,V) \tag{28}$$

$$\Gamma_2(R,V) = R\Gamma_8(V) - \frac{\{J_0(RV)J_0(V)+Y_0(RV)Y_0(V)\}\{J_1(V)Y_0(V)-Y_1(V)J_0(V)\}}{[J_0^2(V)+Y_0^2(V)]^2} - \frac{\{J_1(V)J_0(V)+Y_1(V)Y_0(V)\}\{J_0(RV)Y_0(V)-Y_0(RV)J_0(V)\}}{[J_0^2(V)+Y_0^2(V)]^2} \quad (29)$$

$$\Gamma_3(R,V,Sc) = \frac{J_0(RV/\sqrt{Sc})Y_0(V/\sqrt{Sc})-Y_0(RV/\sqrt{Sc})J_0(V/\sqrt{Sc})}{J_0^2(V/\sqrt{Sc})+Y_0^2(V/\sqrt{Sc})} - \Gamma_5(R,V) \quad (30)$$

$$\Gamma_4(R,V) = \frac{J_0(RV)Y_1(V)-Y_0(RV)J_1(V)}{J_1^2(V)+Y_1^2(V)}, \quad \Gamma_5(R,V) = \frac{\{J_0(RV)Y_0(V)-Y_0(RV)J_0(V)\}}{J_0^2(V)+Y_0^2(V)} \quad (31)$$

$$\Gamma_6(V,Pr) = -\frac{\{J_1(V)J_0(V)+Y_1(V)Y_0(V)\}\{J_1(V/\sqrt{Pr})Y_0(V/\sqrt{Pr})-J_0(V/\sqrt{Pr})Y_1(V/\sqrt{Pr})\}}{\{J_1^2(V)+Y_1^2(V)\}\{J_0^2(V/\sqrt{Pr})+Y_0^2(V/\sqrt{Pr})\}} + \frac{2}{\pi V}\frac{\{J_1(V/\sqrt{Pr})J_0(V/\sqrt{Pr})+Y_1(V/\sqrt{Pr})Y_0(V/\sqrt{Pr})\}}{\{J_1^2(V)+Y_1^2(V)\}\{J_0^2(V/\sqrt{Pr})+Y_0^2(V/\sqrt{Pr})\}} \quad (32)$$

$$\Gamma_7(V,Sc) = \frac{1}{\sqrt{Sc}}\frac{\{J_1(V/\sqrt{Sc})Y_0(V/\sqrt{Sc})-Y_1(V/\sqrt{Sc})J_0(V/\sqrt{Sc})\}}{J_0^2(V/\sqrt{Sc})+Y_0^2(V/\sqrt{Sc})} - \Gamma_8(V) \quad (33)$$

$$\Gamma_8(V) = \frac{\{J_1(V)Y_0(V)-Y_1(V)J_0(V)\}}{J_0^2(V)+Y_0^2(V)} \quad (34)$$

$$\Gamma_9(V) = \frac{\{J_2(V)Y_1(V)-Y_2(V)J_1(V)\}+\{J_1(V)Y_0(V)-Y_1(V)J_0(V)\}}{2\{J_1^2(V)+Y_1^2(V)\}} - \Gamma_8(V) \quad (35)$$

$$\Gamma_{10}(V) = V\frac{\{J_2(V)Y_0(V)-Y_2(V)J_0(V)\}}{2\{J_0^2(V)+Y_0^2(V)\}} - \Gamma_8(V) - 2V\frac{\{J_1(V)J_0(V)+Y_1(V)Y_0(V)\}\{J_1(V)Y_0(V)-Y_1(V)J_0(V)\}}{[J_0^2(V)+Y_0^2(V)]^2} \quad (36)$$

$$\xi(R,V,Pr) = -\frac{\{J_1(V)J_0(V)+Y_1(V)Y_0(V)\}\{J_0(RV/\sqrt{Pr})Y_0(V/\sqrt{Pr})-J_0(V/\sqrt{Pr})Y_0(RV/\sqrt{Pr})\}}{\{J_1^2(V)+Y_1^2(V)\}\{J_0^2(V/\sqrt{Pr})+Y_0^2(V/\sqrt{Pr})\}} + \Gamma_4(R,V) + \frac{2}{\pi V}\frac{\{J_0(RV/\sqrt{Pr})J_0(V/\sqrt{Pr})+Y_0(RV/\sqrt{Pr})Y_0(V/\sqrt{Pr})\}}{\{J_1^2(V)+Y_1^2(V)\}\{J_0^2(V/\sqrt{Pr})+Y_0^2(V/\sqrt{Pr})\}} \quad (37)$$

4. Results and discussions

In order to get an insight into the physics of the problem, the computations of velocity, temperature, concentration, skin-friction and Sherwood number are made for different values of the parameters Pr, Gr, Gc, Sc, t and presented graphically in Figs. 3–13.

Fig. 3 shows the effects of Gr and Gc for $Pr = 0.71$, $t = 1.2$ and $Sc = 0.6$. The thermal Grashof number relates the relative effect of the buoyancy force to the viscous force. The mass Grashof number relates the ratio of the species buoyancy force to the viscous force. The positive values of Gr correspond to cooling of the cylinder by free convection. Heat is therefore conducted away from the vertical cylinder into the fluid, which increases temperature and thereby enhances the buoyancy force. As expected, it is found that an increase in the thermal Grashof number Gr leads to an increase in the velocity. Likewise, the velocity increases with increase in mass Grashof number Gc. In natural convection the buoyancy forces act like a favorable pressure gradient and accelerate fluid and hence the velocity and the boundary-layer thickness increase with the increase in buoyancy parameters. Since the governing equations are coupled together with the buoyancy parameters, the thermal Grashof number as well as mass Grashof number accelerates the fluid and so the velocity and the boundary-layer thickness increase with the increase in Gr or Gc.

Velocity profiles represented by Fig. 4 show the effects of Prandtl number Pr and Schmidt number Sc for $Gr = Gc = 5$ and $t = 1.2$. It can be seen from the figure that the velocity profiles decrease with increase in Prandtl number or Schmidt number. The Prandtl number Pr signifies the relative thickness of the hydrodynamic boundary layer and thermal boundary layer. With increase in Prandtl number the heat capacity of the fluid increases and so gives a negative effect to the thermal

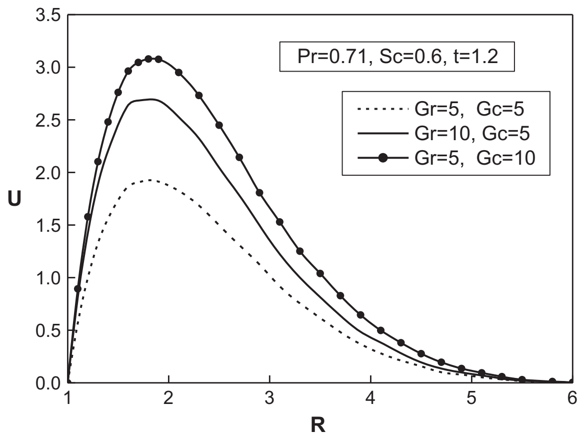

Figure 3 Velocity profiles for $Sc = 0.6$, $t = 1.2$ and $Pr = 0.71$.

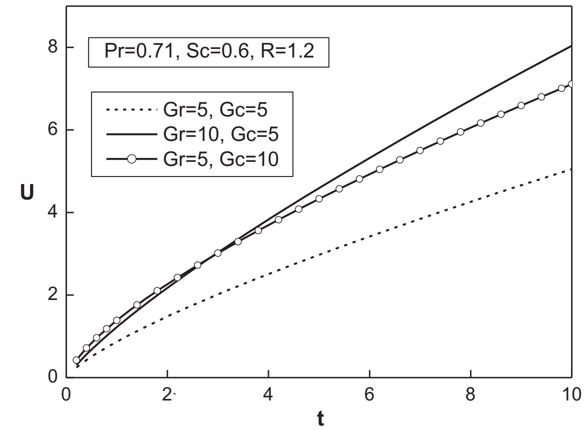

Figure 6 Velocity profiles against t for $Sc = 0.6$, $Pr = 0.71$ and $R = 1.2$.

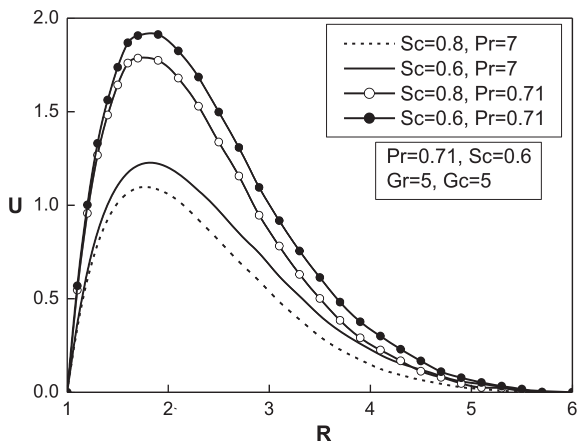

Figure 4 Effects of Sc and Pr on velocity profiles for $Gr = Gc = 5$, $t = 1.2$.

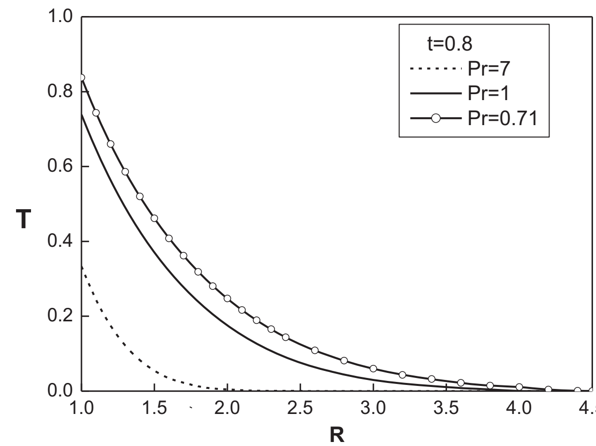

Figure 7 Effect of Pr on temperature profiles at $t = 0.8$.

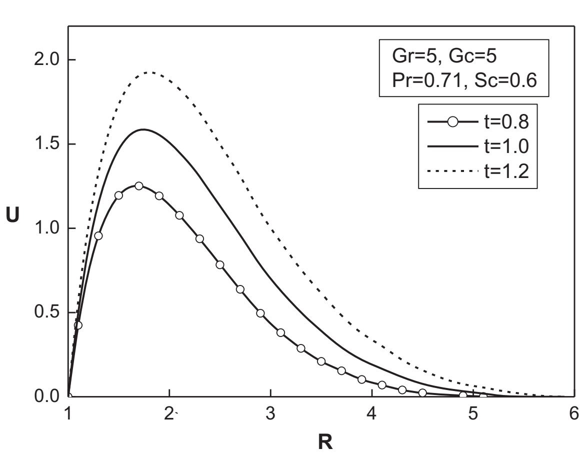

Figure 5 Velocity profiles for $Gr = Gc = 5$, $Pr = 0.71$ and $Sc = 0.6$.

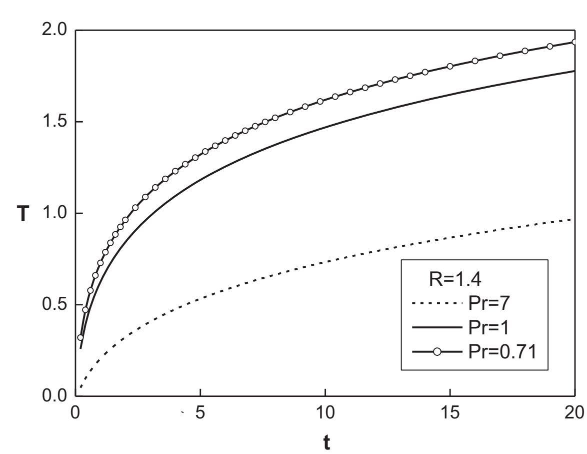

Figure 8 Effect of Pr on temperature profiles against t at $R = 1.4$.

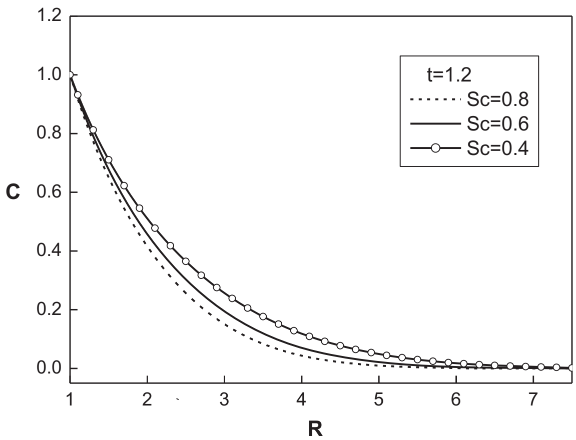

Figure 9 Effects of Sc on concentration profiles at $t = 1.2$.

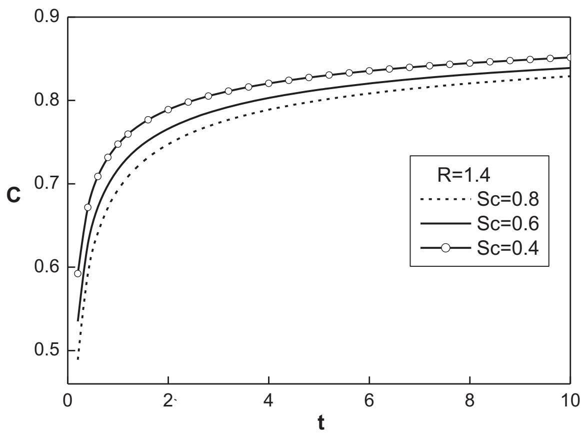

Figure 10 Effects of Sc on concentration profiles at $R = 1.4$.

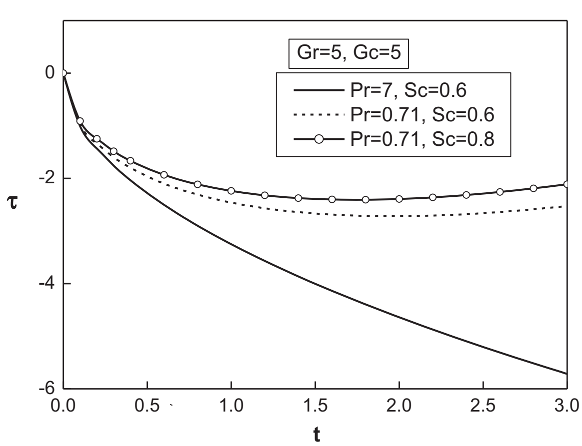

Figure 11 Effects of Pr and Sc on skin friction for $Gr = Gc = 5$.

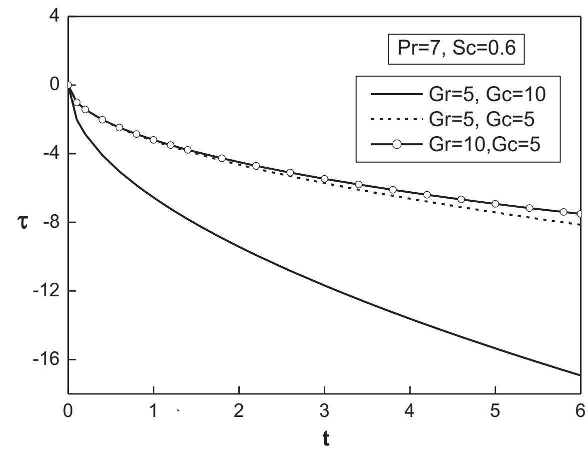

Figure 12 Effects of Gr and Gc on skin friction for $Pr = 7$ and $Sc = 0.6$.

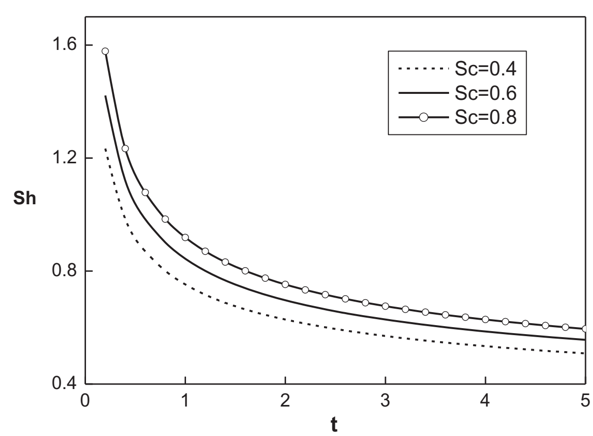

Figure 13 Effects of Sc on Sherwood number.

buoyancy force which causes the decrease in fluid velocity. The Schmidt number Sc signifies the relative thickness of the hydrodynamic boundary layer and concentration boundary layer. Sc determines the diffusion of species to the fluid. With the increase in Schmidt number, the kinematic viscosity increases which leads the velocity to decrease.

Fig. 5 represents the velocity profiles for $Gr = Gc = 5$, $Pr = 0.71$ and $Sc = 0.6$ for different time t. This shows that velocity increases with time. Fig. 6 depicts the velocity profiles against time at $Pr = 0.71$, $Sc = 0.6$ at a radial distance $R = 1.2$ for different values of Gr and Gc. It is observed that for $Pr = 0.71$, the velocity increases unboundedly with time and steady state is not reached. However, for $Pr = 7.0$, the steady state is expected for larger time.

The transient temperature profiles for various Pr are depicted in Fig. 7. It is observed that the thermal boundary layer thickness decreases as the Prandtl number increases from 0.71 (Air) to 7 (Water) due to a decrease in the fluid thermal diffusivity. This is because of the physical fact that, at lower Prandtl number, the fluid has a thicker thermal boundary layer and this decreases the gradient of temperature. The behavior of temperature profiles with respect to time is represented by Fig. 8. The characteristic of the temperature with progressive time is similar to the velocity profiles. The transient concentration profiles for different Sc at $t = 1.2$ are shown in Fig. 9. Fig. 10 shows the concentration profiles for various Sc against time. It is observed that the concentration decreases with increases in the value of Sc. It is remarkable to note that the concentration approaches steady state as time progresses. This is in agreement with Ganesan and Rani [28] and Ganesan and Loganathan [30].

Effects of Pr and Sc on skin friction against time are presented in Fig. 11, where from it is observed that the skin friction monotonically decreases in case of water. The skin friction assumes less negative values in case of air. Furthermore, the possibility of separation of flow can be expected in case of air as time progresses. Similar trend is seen with Gr and Gc against time from Fig. 12. Also, it is seen that an increase in Gr leads to an increase in the skin friction but reverse behavior is seen with Gc. Sherwood number for different values of Schmidt number is shown in Fig. 13, and it can be seen that Sherwood number decreases monotonically with Sc and time. As time progresses, the skin friction is expected to reach steady state for fluids with $Pr = 7.0$ or higher.

5. Conclusions

In this work, we have presented an exact analysis for an unsteady free convective flow past a vertical cylinder with heat and mass transfer under the effect of constant heat flux at the surface of the cylinder. The velocity, temperature and concentration profiles are drawn for different values of parameters such as Pr, Sc and t. Furthermore, we have displayed the effects of different parameters on the skin friction and Sherwood number. Attempt has been made to see whether the flow characteristics reach steady state at larger times. It is observed that the velocity increases with an increase in Gr and Gc, but decreases with Sc. The temperature decreases with the increase in Pr and concentration decreases with Sc. It is remarkable that concentration approaches steady state at larger times. In addition, skin friction increases with Gr and decreases with Sc. On the other hand the Sherwood number decreases monotonically with Sc. As time progresses the skin friction decreases monotonically with Gr. The flow past vertical cylinders with heat and mass transfer have possible application in the emergency cooling of the core of a nuclear reactor in the case of pump or power failure. Finally, the analytical solutions obtained by the Laplace transformation technique in terms of Bessel functions can be applied for validating numerical convection models.

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